Int. J. Heat Mass Transfer. Vol. 33, No. 8, pp. 1759–1766, 1990
Printed in Great Britain

0017-9310/90 $3.00+0.00
Pergamon Press plc

Equivalent conductivity of a heterogeneous medium

K. MURALIDHAR
Department of Mechanical Engineering, Indian Institute of Technology, Kanpur 208016, U.P., India

(*Received* 24 *August* 1988 *and in final form* 18 *May* 1989)

Abstract—The thermal conductivity of a composite medium consisting of a conducting matrix and insulating fibres is studied. The equivalent transverse conductivity of the composite as a function of the fibre volume fraction is of interest. This is obtained in the present work by subjecting an inhomogeneous region to both steady and steady-periodic temperature differences. The resulting set of equations is solved by a finite element technique. Results show that a composite medium can be homogenized using statically determined conductivities, even for unsteady problems.

INTRODUCTION

INHOMOGENEOUS materials are widely used in engineering practice. They have found application in structural elements due to their high strength-to-weight ratio. In several demanding applications such as aircraft wings, satellites, cylinder blocks of IC engines, etc., the composite structure is required to withstand a large thermal loading as well. In some instances, the components which make up the composite have widely differing strengths as well as thermal conductivities. Hence, while the strength is improved, the equivalent conductivity of the composite deteriorates to lower values. For a given heat flux, this can mean higher temperatures in the structure and consequently a lowering of the strength itself. Determination of the effective conductivity of a composite forms the topic of this paper.

A list of formulae useful in calculating thermal conductivity of a fibre composite has been given by Chawla [1]. Baker-Jarvis and Inguva [2] have studied steady heat conduction in a region containing inclusions, by modifying the Laplace equation to account for the microstructure. Hatta and Taya [3] have extended Eshelby's equivalent inclusion method in elasticity [4] to determine the effective conductivity of a composite with highly conducting short fibres randomly oriented within it. Parang *et al.* [5] have studied heat conduction in a region which contains coolant tubes normal to it.

The present work deals with heat conduction in a region containing insulating fibres distributed uniformly within it. This configuration models a metal–matrix composite with low conductivity fibres inserted to improve its strength. It is of interest to determine the extent to which the conductivity changes as a function of the fibre volume fraction. This study is restricted to transverse conductivity alone, since it is clear that longitudinal conductivity is well modelled by the rule of mixtures [1]. The problem of a composite undergoing a transient heating process is also studied here. There is no reason to expect that the static effective conductivities would be applicable for the unsteady problem, since the physical process in each case is different.

Results obtained in this study are equally valid for the important problem of ground water flow through fractured rocks. This extension is possible by identifying temperature with pressure, conductivity with permeability and thermal capacity with storage capacity. The transient conduction problem discussed in this paper also has applications in the hydrofracturing of oil-bearing rocks. However, the problem formulation in this paper is in terms of temperature alone.

FORMULATION

The geometry and the coordinate system considered in this work are given in Fig. 1. The region is taken as square, with a distribution of circular inhomogeneities whose conductivity is approximated as zero. The matrix conductivity is finite, and the conductivity of the composite is normalized with respect to it. The effective conductivity of a composite is defined as that value for an equivalent homogeneous region, which, for a given temperature drop, permits the same amount of energy through it. Since the fibres and the side walls are taken as insulating surfaces, the energy supplied to the composite at $x = 0$ is equal to the energy leaving it, at $x = L$, under steady conditions. The transverse equivalent conductivity is then defined as

$$k_0 = -L\frac{\partial T}{\partial x}\bigg|_{x=0} = -L\frac{\partial T}{\partial x}\bigg|_{x=L} \quad (1)$$

averaged over $y = 0$–L and normalized with respect to the matrix conductivity. For the unsteady problem, part of the energy supplied at $x = 0$ is absorbed by

NOMENCLATURE

A, B, C, D constants of integration in equation (14)
F shape function
Fo Fourier number, $\alpha/\omega L^2$
i $\sqrt{-1}$, imaginary unit
k equivalent thermal conductivity of a composite medium in a generalized transient problem, normalized by k_m
k_0 equivalent thermal conductivity of a composite medium at steady state, normalized by k_m
k_f thermal conductivity of the inhomogeneities embedded in the matrix
k_m matrix thermal conductivity (also characteristic conductivity)
K_1, K_2 components of the element stiffness matrix arising in the FEM
L edge of the square region, normalized by a typical size of the inclusion
NOB number of circular voids, or fibres
Re ( ), Im ( ) real and imaginary parts of a complex quantity
t time
Δt excess time required for a composite to become conducting, over a homogeneous region
T temperature
$\hat{T}$ complex temperature, $T_r + iT_i$
V volume fraction of inclusions in matrix
x, y Cartesian coordinates.

Greek symbols
α thermal diffusivity of matrix, $k_m/(\rho c_p)_m$
δ thermal boundary-layer thickness on the heated edge
λ_n nth eigenvalue in equation (18)
ω $\bar{\omega}/\alpha$
$\bar{\omega}$ forcing frequency of thermal loading.

Other symbol
$\underset{\sim}{\nabla}$ gradient operator, $(\partial/\partial x, \partial/\partial y)$.

Subscripts
i, j ith row, jth column element of matrix
x, y, t, n $\partial/\partial x$, $\partial/\partial y$, $\partial/\partial t$, $\partial/\partial n$, respectively.

the matrix to raise its temperature. Hence the energy leaving the region at $x = L$ is less than the energy supplied. In applications such as the cylinder block of an IC engine, it is of interest to determine the time taken for the region to become conducting, i.e. for the heat flux at $x = L$ to be a significant portion of the heat flux at $x = 0$. Hence, for transient problems studied in this work, we define

$$k = -L\frac{\partial T}{\partial x}\bigg|_{x=L} \tag{2}$$

averaged over $y = 0$–L. Clearly, for steady conduction with no inhomogeneities, k_0 is unity. It is less than unity when the fibre volume fraction is non-zero and for transient problems. Equations (1) and (2) are in dimensionless form. The temperature T is normalized by the imposed temperature difference across the composite region. Factor L arises from the heat flux expression for a homogeneous region of size L, subject to a unit temperature drop ($=k_m/L$). Steady heat conduction is governed by the Laplace equation

$$\nabla^2 T = T_{xx} + T_{yy} = 0 \text{ in the matrix} \tag{3}$$

with the boundary conditions

$$x = 0, \quad T = 1$$
$$x = L, \quad T = 0$$
$$y = 0, L, \quad T_y = 0. \tag{4}$$

On all fibre surfaces, $T_n = 0$, where n is the unit outward drawn normal on these surfaces. This is based on the assumption that the fibre conductivity is very small compared to the matrix conductivity. The insulated side wall boundary conditions at $y = 0$ and L may be interpreted as symmetry conditions in a large composite, which has a repeating pattern of fibre distribution normal to the mean temperature gradient.

The unsteady problem is governed by

$$\nabla^2 T = \frac{1}{\alpha} T_t \tag{5}$$

the boundary conditions given by equation (4) and an initial condition

$$t = 0, \quad T = 0 \text{ in the matrix.} \tag{6}$$

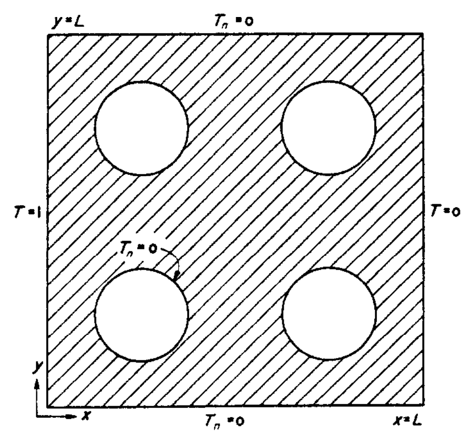

FIG. 1. Physical domain, coordinate system and boundary condition.

In equation (5), α is the thermal diffusivity in the matrix.

For the transient problem, the application of equation (2) leads to an effective conductivity whose dependence on time changes with the choice of the initial conditions. To keep the analysis general, the temperature difference across the composite is taken as

$$T(x=0)-T(x=L) = e^{i\bar{\omega}t} \tag{7}$$

and dynamic steady state is allowed to be reached. Then the local temperature is expressed as

$$T(x,y,t) = \hat{T}\, e^{i\bar{\omega}t} \tag{8a}$$

where

$$\hat{T} = T_r(x,y,\bar{\omega}) + iT_i(x,y,\bar{\omega}) \tag{8b}$$

and i is the imaginary unit. The determination of T is now reduced to the computation of the complex temperature $\hat{T}$. This process of moving from the time domain to the frequency domain can be interpreted as a Laplace transform. Specific solutions of transient problems can be obtained by inverting this transform. This is usually performed by numerical methods.

The time taken to reach dynamic steady state is not relevant if the analysis given below is employed to construct the inverse Laplace transform. However, the dynamic steady state may be of direct interest in some applications. The time taken to reach this steady state depends on δ, the thickness of the thermal boundary layer. As shown later in this paper (equation (20)), the penetration depth L is related to the largest frequency which can cover this distance as $\omega \sim 1/L^2$. Hence large frequencies would cover small distances and reach steady state rapidly.

The real and imaginary components of complex temperature T can be interpreted as follows. Since the physical temperature T is

$$\begin{aligned} T &= \mathrm{Re}\,[\hat{T}\, e^{i\omega t}] \\ &= T_r \cos \omega t - T_i \sin \omega t \end{aligned}$$

and $T_i = 0$ at $x = 0$, T_r is the in-phase part of the temperature relative to the imposed overall temperature difference and T_i the out-of-phase part.

Equations (8), introduced in equation (5) with the boundary conditions, give rise to the following problem, in terms of T_r and T_i:

$$\begin{aligned} \nabla^2 T_r + \omega T_i &= 0 \\ \nabla^2 T_i - \omega T_r &= 0 \end{aligned} \tag{9}$$

where $\omega = \bar{\omega}/\alpha$. The boundary conditions are

$$\begin{aligned} x = 0, \quad T_r = 1, \quad T_i = 0 \\ x = L, \quad T_r = 0, \quad T_i = 0 \end{aligned} \tag{10}$$

and the normal gradients T_{rn} and $T_{in} = 0$ on all other boundaries. Equations (9) are coupled and have to be solved simultaneously. The equivalent conductivity is a complex quantity defined as

$$k(\omega) = -L\left[\frac{\partial T_r}{\partial x} + i\frac{\partial T_i}{\partial x}\right]_{x=L}. \tag{11}$$

Its real and imaginary parts must both be fully determined to calculate the transient temperature distribution through a Laplace inversion process.

The void fraction (i.e. the fraction of fibres in a composite, or the percentage contact area in a rock fracture) ranges from 0 to 20% in this work.

METHOD OF SOLUTION

Equations (3) and (4) and (9) and (10) have been solved by a Galerkin finite element method using six-noded isoparametric elements [6]. At each node, we solve for both T_r and T_i. The circular shape of the inhomogeneities is exactly represented in the calculation. The element matrix structure here is of the form

$$\begin{bmatrix} [K_1] & \omega[K_2] \\ -\omega[K_2] & [K_1] \end{bmatrix} \begin{bmatrix} T_r \\ T_i \end{bmatrix} = 0 \tag{12}$$

where $K_{1ij} = \int (F_{ix}F_{jx} + F_{iy}F_{jy})\, d\Omega$, $K_{2ij} = \int F_i F_j\, d\Omega$ and $i, j = 1$–6. $[K_1]$ arises from Galerkin integration of the terms $\nabla^2 T_r$ and $\nabla^2 T_i$ in equation (9). $[K_2]$ arises from the coupling terms T_i and T_r in equation (9). Matrix inversion is accomplished by a sparse matrix solver [7]. While solving equation (9), the matrix formed by the FEM becomes increasingly unsymmetric for large values of ω. It is also ill conditioned due to the large off-diagonal terms. Fortunately, $|k(\omega)| \to 0$ as $\omega \to \infty$, and the calculation can be terminated at a finite forcing frequency. For large ω, the solution of equation (9) is $T_r = T_i = 0$ in the bulk of the solid. The mean temperature field then fails to satisfy the boundary condition at $x = 0$. Matrix ill-conditioning signals this change in physical phenomena in the region being studied.

The finite element code has been well tested for both the steady and steady-periodic problems. Analytical solutions are possible for a homogeneous material. The computer program has been tested by reproducing results given by Ozisik [8] for a steady temperature difference, and a variety of side wall conditions. For the time periodic problem, equation (9) can be combined to form

$$\nabla^2\nabla^2 T_r + \omega^2 T_r = 0 \tag{13a}$$

with the boundary conditions

$$\begin{aligned} x = 0, \quad T_r = 1, \quad (T_r)_{xx} = 0 \\ x = L, \quad T_r = 0, \quad (T_r)_{xx} = 0. \end{aligned} \tag{13b}$$

This system of equations, in one dimension, has an analytical solution

$$T_r = A\, e^{cx} \sin cx + B\, e^{-cx} \sin cx + C\, e^{cx} \cos cx + D\, e^{-cx} \cos cx \tag{14}$$

where A, B, C and D are integration constants to be obtained from equation (13b), and $c = \sqrt{(\omega/2)}$. The results of the finite element code match this solution within 1%.

The number of elements is so chosen that each fibre surface is represented by 16 nodes. This ensures that the energy balance between inlet and exit ($x = 0$ and L) is within 0.01% for the steady problem. The same discretization has also been used for the unsteady problem.

A region size of $L = 6$ has been used in the present work. The fibre diameter is of order unity. The effective conductivity of the composite under static thermal loading conditions is insensitive to the choice of L, and depends primarily on the fibre volume fraction. This is discussed further in the next section. Under dynamic loading conditions, the conductivity is a strong function of L. This is because, at each value of ω, a thermal boundary-layer is formed, whose thickness is $\delta(\omega)$. If $\delta \leqslant L$, the conductivity, defined in this study at the energy exit plane, is zero. It becomes non-zero only for values of $\delta > L$. The frequency corresponding to $\delta = L$ is hence a critical value, in the sense that $\omega < \omega_{cr}$ makes the composite conducting, and $\omega > \omega_{cr}$ makes it insulating. This observation has strong implications in the cooling of IC engines. Suppose the thickness of the cylinder block is greater than $\delta(\omega)$, where ω is now the frequency of combustion. Then, no amount of cooling from outside would be of use, since the block is an insulator at that frequency.

Even though k is a function of both ω and L, dimensional analysis shows that it can be made a unique function of the Fourier number, $Fo = 1/\omega L^2 = \alpha/\bar{\omega} L^2$. Hence, the results presented here for $L = 6$ can be extended to any combination of L and ω, by keeping Fo invariant. For a composite, k is also a function of geometry and the scaling (i.e. ωL^2) is not necessarily valid. However, extensive numerical experiments have confirmed that k is a unique function of Fo, and it is sufficient to perform calculations for a single value of L. It will be shown in the next section (Fig. 5) that for both the matrix and the composite, k/k_0 is a unique function of the Fourier number

$$\frac{k}{k_0} = G(Fo). \tag{15}$$

Here k_0 is the equivalent conductivity determined under static conditions.

RESULTS

Figure 2 shows a plot of the static equivalent conductivity k_0 as a function of the fibre volume fraction. Since the fibres are taken as non-conducting, they may as well be thought of as voids distributed in the matrix. The FEM result obtained in this study has been compared to the rule of mixtures

$$k_0 = Vk_m + (1-V)k_f \tag{16}$$

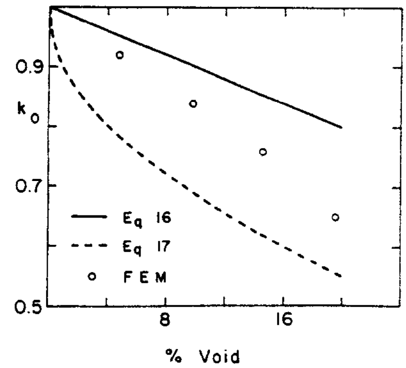

FIG. 2. Plot of static equivalent conductivity as a function of void fraction.

where V is the void fraction and subscripts m and f refer to matrix and fibre, respectively. In this work, $k_f = 0$. FEM results have also been compared to the empirical correlation for transverse conductivity [1, 9]:

$$k_0 = (1-\sqrt{V})k_m + \frac{k_m\sqrt{V}}{1-\sqrt{V}\left(1-\frac{k_m}{k_f}\right)}. \tag{17}$$

Equation (16) is appropriate for longitudinal conductivity, where the fibres are aligned with the direction of the mean temperature gradient. It overpredicts transverse conductivity, and is generally not valid for this problem. Figure 2 also shows that the correlation given in equation (17) underpredicts conductivity with respect to the numerically computed values. In particular, it is very inaccurate for low void fractions, since it shows a very sharp drop in conductivity for $V < 5\%$. Its validity for a large void fraction (greater than 20%) has not been tested against the FEM. This is because the computer memory required to solve such problems is found to be excessive. The best line fit through the numerical data is found to be

$$k_0 = 1 - 1.63V \tag{18}$$

for $0 < V < 0.2$. It predicts zero conductivity for $V \geqslant 0.61$. For such a large proportion of inhomogeneity, it is inappropriate to use a continuum model to determine equivalent properties. The behavior of the composite would be a strong function of the pattern of distribution of the voids (or fibres, or contact areas, as the case may be). The value of k may fluctuate between zero and finite values. The global response of the composite medium can then be characterized only as a statistical average. This is an entirely different problem and is not addressed here.

The effective conductivity of the composite is a function of the pattern of distribution of voids in the matrix. Figure 3 shows the variation in k_0 as a function of this pattern, for fixed values of void percentage. The patterns used are summarized in Table 1. In Fig. 3, NOB refers to the number of circular voids in the

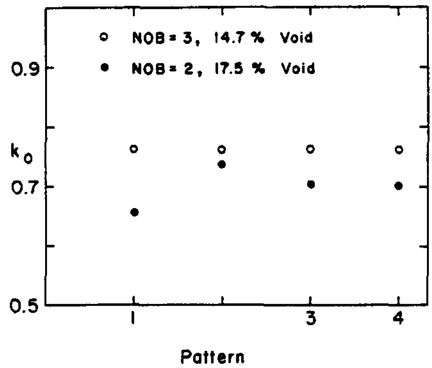

FIG. 3. Plot of static equivalent conductivity as a function of pattern of distribution of voids.

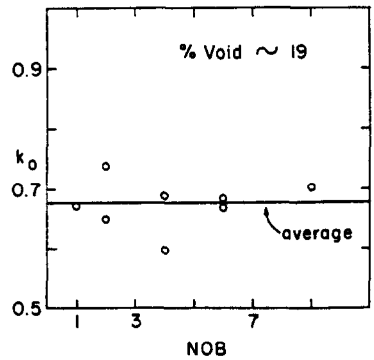

FIG. 4. Plot of static equivalent conductivity as a function of fineness of distribution of voids.

region of interest. The standard deviation of k_0 from its mean value is within 4.3% for both void fractions shown. This mean value has been plotted in Fig. 2 as representative of equivalent conductivity for a given void fraction. The equivalent conductivity is seen to be only weakly dependent on the pattern of distribution of the inhomogeneities, and is primarily a function of their proportion in the conducting zone.

Figure 4 shows a plot of the static equivalent conductivity as a function of fineness of distribution of the voids, for a given total void fraction (= 19% in this figure). For a given size of void, the two patterns which produce extreme conductivities are also displayed in the figure. The scatter is seen to reduce as the size of the void reduces and its number increases to keep the void fraction constant. Hence, it may be concluded that both a fine distribution of inhomogeneities and a coarse one will give rise to nearly the same conductivities.

Figure 5 is a plot of the real part of dynamic equivalent conductivity as a function of forcing frequency ω. The conductivity is normalized by its static value k_0. Curves for both void fractions of 0 and 20% have been plotted. The negative values of Re $[k]$ for a certain range of frequencies should come as no surprise, because k is defined as the dimensionless temperature gradient at the exit plane. Since the inlet temperature (at $x = 0$) fluctuates between $+1$ and -1, and the exit temperature is zero, this gradient is allowed to change sign. The curve for a void fraction of 0% can also be obtained analytically from equation (14). The comparison between the FEM result shown in Fig. 5 and that obtained from equation (14) is found to be quite good.

With k normalized by the static conductivity k_0, it is seen in Fig. 5 that the curves Re $[k/k_0] = f_1(\omega)$ for each void fraction (0 and 20%) nearly overlap each other. It will be assumed here that these curves are in fact identical. Figure 6 shows that the plot of Im $[k/k_0] = f_2(\omega)$ is also nearly unique, independent of the void fraction. Plots of Re $[k]$ and Im $[k]$ for other void fractions between 0 and 20% confirm this trend. Since the steady-periodic problem is a building block for any transient problem (and hence is a generalized transient problem, away from initial conditions), the following conclusion can be arrived at. The exit temperature gradient in a general two-dimensional transient heat conduction problem with inhomogeneities can be obtained from a one-dimensional transient problem by multiplying the frequency-dependent matrix conductivity by the static equivalent conductivity k_0. The exception to this rule occurs when a homogeneous region has a two-dimensional temperature field due to boundary conditions. This case is not studied here.

The implication of the observation given above is the following. To compute the quantity $\partial T/\partial x|_{x=L}$ at various instants of time, and to determine the time taken by a composite region to become conducting, the full problem $T_{xx} + T_{yy} = 1/\alpha T_t$ can be simplified to $T_{xx} = 1/\alpha T_t$. The one-dimensional transient problem has an analytical solution for the following choice of initial and boundary conditions:

Table 1. Distribution of centres of voids in matrix used in Fig. 3

	$NOB = 2$ Radius = 1				$NOB = 3$ Radius = 0.75					
Pattern	x	y	x	y	x	y	x	y	x	y
1	3.0	1.5	3.0	4.5	1.5	1.5	1.5	4.5	4.5	3.0
2	1.5	3.0	4.5	3.0	1.5	3.0	4.5	1.5	4.5	4.5
3	1.5	4.5	4.5	1.5	3.0	4.5	1.5	1.5	4.5	1.5
4	1.5	1.5	4.5	4.5	3.0	1.5	1.5	4.5	4.5	4.5

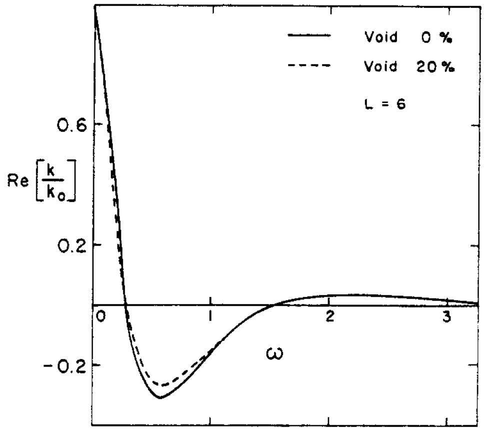

FIG. 5. Real part of dynamic equivalent conductivity normalized by k_0, plotted as a function of the forcing frequency.

$$x = 0 \quad T = 1, \quad t > 0$$

$$x = L \quad T = 0, \quad t > 0$$

$$t = 0 \quad T = 0, \quad 0 \leqslant x \leqslant L.$$

This solution is

$$T(x,t) = \left(1 - \frac{x}{L}\right) - \frac{2}{L}\sum_{n=1}^{\infty}\frac{e^{-\alpha\lambda_n^2 t}}{\lambda_n}\sin\lambda_n x \qquad (19a)$$

where $\lambda_n = n\pi/L$. The dimensionless temperature gradient at the exit plane ($x = L$) of the homogeneous region is

$$k(t) = -1 + 2\sum_{n=1}^{\infty}(-1)^{n+1}e^{-\alpha\lambda_n^2 t}. \qquad (19b)$$

For a composite region, generalized transient analysis shows that

$$\frac{k(t)}{k_0} = -1 + 2\sum_{n=1}^{\infty}(-1)^{n+1}e^{-\alpha\lambda_n^2 t}. \qquad (19c)$$

The time taken by a region with a distribution of voids to become conducting to various extents (5%, 25%, etc.) can be calculated from equation (19c).

From Figs. 5 and 6, it is clear that the dynamic conductivity is reduced to a small magnitude for $\omega > 3$, when $L = 6$. This corresponds to a Fourier number of

$$Fo = \frac{\alpha}{\bar{\omega}L^2} = \frac{1}{\omega L^2} = \frac{1}{108}.$$

Hence, the cut-off frequency for any other region size, homogeneous or otherwise, can be calculated from

$$\bar{\omega}_{\text{cut-off}} = \frac{108\alpha}{L^2}. \qquad (20)$$

This relationship has application in the design of cylinder blocks for IC engines, which are subject to thermal cyclic loading. For a loading frequency greater than the cut-off value given by equation (20), the mean energy conducted through the material drops to zero. The cylinder block will then have to be cooled by having coolant channels in its interior.

Figure 7 is a plot of the dimensionless time taken by a composite region to become conducting as a function of the void percentage, obtained from equation (19). This time is independent of the length L, being defined as $\alpha\pi^2 t/L^2$, where α is the matrix diffusivity and t is the real time. The numbers shown in the graph (5%, 25% etc.) refer to the criteria used to determine whether the piece is conducting or not. For example, with a 5% criterion, this time is what is required for k/k_0 to be just equal to 0.05, starting from a zero value. For a 5% criterion, a composite region with 20% inhomogeneities takes 42% longer to become conducting, relative to a uniform matrix alone. This figure is 42% for the 95% criterion as well. Hence, the excess time taken by a material to become conducting is independent of the criterion used and is a unique function of the void fraction. This observation has been tested for all void percentages, between 0 and 20%, and has been found to be valid. The plot of this excess time Δt versus the void fraction V is shown in Fig. 8.

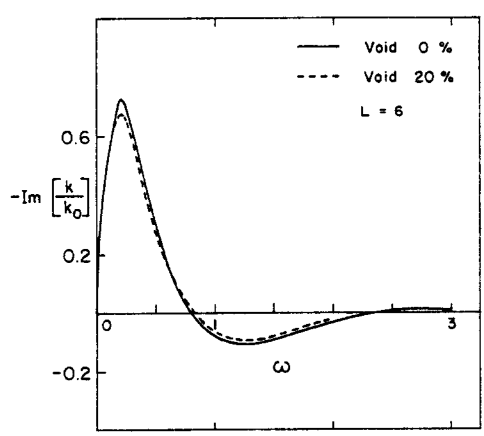

FIG. 6. Imaginary part of dynamic equivalent conductivity normalized by k_0, plotted as a function of forcing frequency.

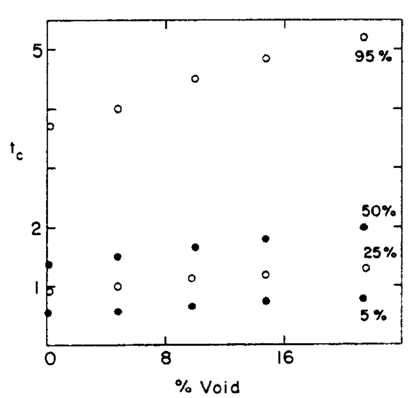

FIG. 7. Plot of time taken for a composite to become conducting as a function of the void fraction.

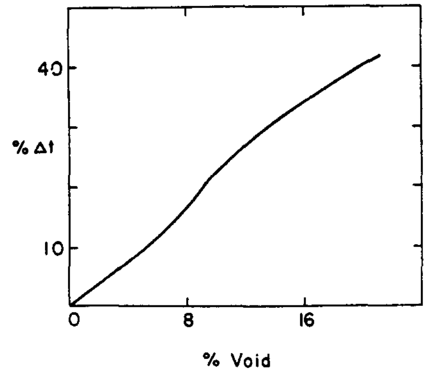

FIG. 8. Plot of excess time taken by a composite to become conducting over a homogeneous region, as a function of the void fraction.

CONCLUSIONS

The following are the conclusions arrived at in this work.

1. The static equivalent conductivity of a composite region is primarily a function of the percentage of inhomogeneities. It depends weakly on the pattern of distribution and fineness of their size. Based on Fig. 2, the best fit line through the numerical data is

$$k_0 = 1-1.63V, \quad 0 \leqslant V \leqslant 0.2.$$

2. The real and imaginary parts of the equivalent conductivity, calculated at the exit plane of a square composite region of length L, have the following properties.

(i) Re $[k/k_0]$ and Im $[k/k_0]$ have a unique value corresponding to the dimensionless parameter, $\alpha/\omega L^2$.

(ii) They reduce to zero for $\alpha/\omega L^2 < 1/108$.

(iii) In view of conclusion 2(i), the calculation of the transient exit temperature gradient for a two-dimensional composite region can be obtained from a one-dimensional homogenized region. This invariably has an analytical solution. The effective conductivity of the composite is the product of the static conductivity k_0 and the transient conductivity of the matrix alone (see equation (19c)).

(iv) The excess time taken by a cold composite to become conducting, over and above that for a homogeneous region, is independent of the criterion used to determine it. It is a function of the void fraction V only.

Acknowledgement—Calculations for this work were carried out at the CAD Centre, IIT, Kanpur.

REFERENCES

1. K. K. Chawla, *Composite Materials*. Springer, New York (1987).
2. J. Baker-Jarvis and R. Inguva, Heat conduction in heterogeneous materials, *Trans. ASME, J. Heat Transfer* **107**, 39–43 (1985).
3. H. Hatta and M. Taya, Effective thermal conductivity of a misoriented short fiber composite, *J. Appl. Phys.* **58**, 2478–2486 (1985).
4. J. D. Eshelby, The determination of the elastic field of an ellipsoidal inclusion, and related problems, *Proc. R. Soc. Lond.* **A241**, 376–396 (1957).
5. M. Parang, R. V. Arimilli and S. P. Ketkar, Optimal positioning of tubes in arbitrary two dimensional regions using a special boundary integral method, *Trans. ASME, J. Heat Transfer* **109**, 826–830 (1987).
6. O. C. Zienkiewivz, *The Finite Element Method*. McGraw-Hill, New York (1977).
7. I. S. Duff, MA 28—a set of Fortran subroutines for sparse, unsymmetric linear equations. AERE, Harwell (1980).
8. M. N. Ozisik, *Heat Conduction*. Wiley, New York (1980).
9. C. C. Chamis, NASA Tech. Memo 83320 (1983).

CONDUCTIVITE EQUIVALENTE D'UN MILIEU HETEROGENE

Résumé—On étudie la conductivité thermique d'un milieu composite formé d'une matrice conductrice et de fibres isolantes. On s'intéresse à la conductivité transverse équivalente du composite en fonction de la fraction de volume de fibre. Elle est obtenue en soumettant une région du milieu à des différences de température soit stables, soit périodique établies. Le système d'équations résultant est résolu par une technique d'éléments finis. Les résultats montrent qu'un milieu composite peut être homogénéisé en utilisant des conductivités statistiquement déterminées, même pour des problèmes non permanents.

EFFEKTIVE WÄRMELEITFÄHIGKEIT EINES HETEROGENEN MEDIUMS

Zusammenfassung—Die Wärmeleitfähigkeit eines zusammengesetzten Mediums aus einer leitenden Matrix und isolierenden Fasern wird untersucht. Die effektive quergerichtete Leitfähigkeit des Materials wird in Abhängigkeit vom Volumenanteil der Fasern betrachtet. In dieser Arbeit wird ein inhomogenes Gebeit sowohl stationären als auch periodisch stationären Temperaturdifferenzen unterworfen. Es ergibt sich ein Satz von Gleichungen, der mit Hilfe der Finite-Elemente-Technik gelöst wird. Die Ergebnisse zeigen, daß ein derartiges zusammengesetztes Medium als homogen betrachtet werden kann, wenn stationär bestimmte Leitfähigkeiten verwendet werden—selbst bei instationären Problemen.

ЭКВИВАЛЕНТНАЯ ПРОВОДИМОСТЬ НЕОДНОРОДНОЙ СРЕДЫ

Аннотация—Исследуется теплопроводность композитной среды, состоящей из проводящей матрицы с непроводящими волокнами. Особое внимание уделяется эквивалентной поперечной проводимости композитной среды как функции объемной доли волокон. В настоящей работе она определяется с помощью создания в неоднородной области стационарной и стационарно-периодической разностей температур. Полученная система уравнений решается методом конечных элементов. Результаты показывают, что композитная среда может рассматриваться как гомогенизированная с использованием статистически определенных величин теплопороводности даже для нестационарных задач.